

# Mark Scheme (Results)

## January 2007

GCE

GCE Mathematics

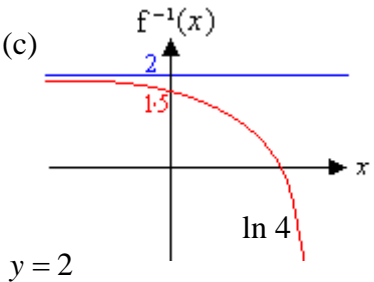
Core Mathematics C3 (6665)

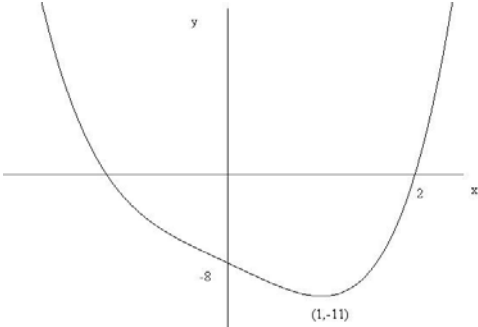
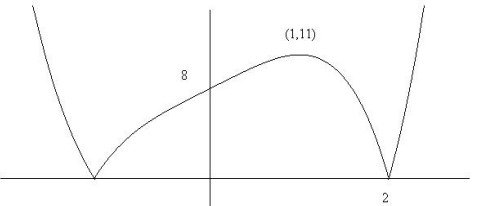
**January 2007**  
**6665 Core Mathematics C3**  
**Mark Scheme**

Question Number	Scheme	Marks
1.	<p>(a) <math>\sin 3\theta = \sin(2\theta + \theta) = \sin 2\theta \cos \theta + \cos 2\theta \sin \theta</math>  <math>= 2 \sin \theta \cos^2 \theta + (1 - 2 \sin^2 \theta) \sin \theta</math>  <math>= 2 \sin \theta - 2 \sin^3 \theta + \sin \theta - 2 \sin^3 \theta</math>  <math>= 3 \sin \theta - 4 \sin^3 \theta</math> *</p> <p>(b) <math>\sin 3\theta = 3 \times \frac{\sqrt{3}}{4} - 4 \left( \frac{\sqrt{3}}{4} \right)^3 = \frac{3\sqrt{3}}{4} - \frac{3\sqrt{3}}{16} = \frac{9\sqrt{3}}{16}</math></p> <p>equivalent</p>	<p>B1 B1 B1 M1 A1 (5)</p> <p>or exact M1 A1 (2)</p> <p>[7]</p>
2.	<p>(a) <math>f(x) = \frac{(x+2)^2, -3(x+2)+3}{(x+2)^2}</math>  <math>= \frac{x^2 + 4x + 4 - 3x - 6 + 3}{(x+2)^2} = \frac{x^2 + x + 1}{(x+2)^2}</math> *</p> <p>(b) <math>x^2 + x + 1 = \left(x + \frac{1}{2}\right)^2 + \frac{3}{4}, &gt; 0</math> for all values of <math>x</math>.</p> <p>(c) <math>f(x) = \frac{\left(x + \frac{1}{2}\right)^2 + \frac{3}{4}}{(x+2)^2}</math></p> <p>Numerator is positive from (b)  <math>x \neq -2 \Rightarrow (x+2)^2 &gt; 0</math> (Denominator is positive)  Hence <math>f(x) &gt; 0</math></p>	<p>M1 A1, A1 A1 (4)</p> <p>M1 A1, A1 (3)</p> <p>B1 (1)</p> <p>[8]</p>
	<p><i>Alternative to (b)</i></p> $\frac{d}{dx}(x^2 + x + 1) = 2x + 1 = 0 \Rightarrow x = -\frac{1}{2} \Rightarrow x^2 + x + 1 = \frac{3}{4}$ <p>A parabola with positive coefficient of <math>x^2</math> has a minimum <math>\Rightarrow x^2 + x + 1 &gt; 0</math>  Accept equivalent arguments</p>	<p>M1 A1 A1 (3)</p>

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3.	<p>(a) <math>y = \frac{\pi}{4} \Rightarrow x = 2 \sin \frac{\pi}{4} = 2 \times \frac{1}{\sqrt{2}} = \sqrt{2} \Rightarrow P \in C</math></p> <p>Accept equivalent (reversed) arguments. In any method it must be clear that <math>\sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}</math> or exact equivalent is used.</p> <p>(b) <math>\frac{dx}{dy} = 2 \cos y \quad \text{or} \quad 1 = 2 \cos y \frac{dy}{dx}</math></p> <p><math>\frac{dy}{dx} = \frac{1}{2 \cos y}</math>      May be awarded after substitution</p> <p><math>y = \frac{\pi}{4} \Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{2}} \quad *</math>      cso</p> <p>(c) <math>m' = -\sqrt{2}</math></p> <p><math>y - \frac{\pi}{4} = -\sqrt{2}(x - \sqrt{2})</math></p> <p><math>y = -\sqrt{2}x + 2 + \frac{\pi}{4}</math></p>	<p>B1      <b>(1)</b></p> <p>M1 A1</p> <p>M1</p> <p>A1      <b>(4)</b></p> <p>B1</p> <p>M1 A1</p> <p>A1      <b>(4)</b></p> <p><b>[9]</b></p>
4.	<p>(i) <math>\frac{dy}{dx} = \frac{(9+x^2) - x(2x)}{(9+x^2)^2} \left( = \frac{9-x^2}{(9+x^2)^2} \right)</math></p> <p><math>\frac{dy}{dx} = 0 \Rightarrow 9 - x^2 = 0 \Rightarrow x = \pm 3</math></p> <p><math>\left(3, \frac{1}{6}\right), \left(-3, -\frac{1}{6}\right)</math>      Final two A marks depend on second M only</p> <p>(ii) <math>\frac{dy}{dx} = \frac{3}{2}(1+e^{2x})^{\frac{1}{2}} \times 2e^{2x}</math></p> <p><math>x = \frac{1}{2} \ln 3 \Rightarrow \frac{dy}{dx} = \frac{3}{2}(1+e^{\ln 3})^{\frac{1}{2}} \times 2e^{\ln 3} = 3 \times 4^{\frac{1}{2}} \times 3 = 18</math></p>	<p>M1 A1</p> <p>M1 A1</p> <p>A1, A1      <b>(6)</b></p> <p>M1 A1 A1</p> <p>M1 A1      <b>(5)</b></p> <p><b>[11]</b></p>

Question Number	Scheme	Marks
5.	<p>(a) <math>R^2 = (\sqrt{3})^2 + 1^2 \Rightarrow R = 2</math>  <math>\tan \alpha = \sqrt{3} \Rightarrow \alpha = \frac{\pi}{3}</math></p> <p>(b) <math>\sin(x + \text{their } \alpha) = \frac{1}{2}</math>  <math>x + \text{their } \alpha = \frac{\pi}{6} \left( \frac{5\pi}{6}, \frac{13\pi}{6} \right)</math>  <math>x = \frac{\pi}{2}, \frac{11\pi}{6}</math></p> <p>The use of degrees loses only one mark in this question. Penalise the first time it occurs in an answer and then ignore.</p>	<p>M1 A1  M1 A1 (4)  M1  A1  M1 A1 (4)  [8]</p>

Question Number	Scheme	Marks
6.	<p>(a) <math>y = \ln(4 - 2x)</math>  <math>e^y = 4 - 2x</math> leading to <math>x = 2 - \frac{1}{2}e^y</math> Changing subject and removing <math>\ln</math>  <math>y = 2 - \frac{1}{2}e^x \Rightarrow f^{-1} \mapsto 2 - \frac{1}{2}e^x</math> *                      Domain of <math>f^{-1}</math> is</p> <p>(b) Range of <math>f^{-1}</math> is <math>f^{-1}(x) &lt; 2</math> (and <math>f^{-1}(x) \in \mathbb{R}</math>)</p> <p>(c) </p> <p>(d) <math>x_1 \approx -0.3704, x_2 \approx -0.3452</math>                      If more than 4 dp given in this part a maximum on one mark is lost.                      Penalise on the first occasion.</p> <p>(e) <math>x_3 = -0.354\ 030\ 19 \dots</math>  <math>x_4 = -0.350\ 926\ 88 \dots</math>  <math>x_5 = -0.352\ 017\ 61 \dots</math>  <math>x_6 = -0.351\ 633\ 86 \dots</math> Calculating to at least <math>x_6</math> to at least four dp  <math>k \approx -0.352</math></p> <p>Alternative to (e)  <math>k \approx -0.352</math> Found in any way                      Let <math>g(x) = x + \frac{1}{2}e^x</math>  <math>g(-0.3515) \approx +0.0003, g(-0.3525) \approx -0.001</math>                      Change of sign (and continuity) <math>\Rightarrow k \in (-0.3525, -0.3515)</math>  <math>\Rightarrow k = -0.352</math> (to 3 dp)</p>	<p>M1 A1                      A1                      B1 (4)                      B1 (1)                      Shape                      1.5  <math>\ln 4</math>                      B1                      B1 (4)                      B1, B1 (2)                      M1                      A1 (2)                      [13]                      M1                      A1 (2)</p>

Question Number	Scheme	Marks
7.	<p>(a) <math>f(-2) = 16 + 8 - 8 (= 16) &gt; 0</math>  <math>f(-1) = 1 + 4 - 8 (= -3) &lt; 0</math>                      Change of sign (and continuity) <math>\Rightarrow</math> root in interval <math>(-2, -1)</math>                      ft their calculation as long as there is a sign change</p> <p>(b) <math>\frac{dy}{dx} = 4x^3 - 4 = 0 \Rightarrow x = 1</math>                      Turning point is <math>(1, -11)</math></p> <p>(c) <math>a = 2, b = 4, c = 4</math></p> <p>(d) </p> <p>(e) </p>	<p>B1                      B1                      B1ft <b>(3)</b></p> <p>M1 A1                      A1 <b>(3)</b></p> <p>B1 B1 B1 <b>(3)</b></p> <p>Shape                      ft their turning point in                      correct quadrant only                      2 and -8                      B1 <b>(3)</b></p> <p>Shape                      B1 <b>(1)</b>  <b>[13]</b></p>

Question Number	Scheme	Marks
8.	<p>(i) <math>\sec^2 x - \operatorname{cosec}^2 x = (1 + \tan^2 x) - (1 + \cot^2 x)</math>  <math>= \tan^2 x - \cot^2 x</math> *</p> <p>(ii)(a) <math>y = \arccos x \Rightarrow x = \cos y</math>  <math>x = \sin\left(\frac{\pi}{2} - y\right) \Rightarrow \arcsin x = \frac{\pi}{2} - y</math>                      Accept  <math>\arcsin x = \arcsin \cos y</math></p> <p>(b) <math>\arccos x + \arcsin x = y + \frac{\pi}{2} - y = \frac{\pi}{2}</math></p>	<p>M1 A1                      A1 (3)                      B1                      B1 (2)                      B1 (1)                      [6]</p>
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	<p><i>Alternatives for (i)</i></p> <p><math>\sec^2 x - \tan^2 x = 1 = \operatorname{cosec}^2 x - \cot^2 x</math>                      Rearranging <math>\sec^2 x - \operatorname{cosec}^2 x = \tan^2 x - \cot^2 x</math> *                      cso</p> <p><math>\left( \text{LHS} = \frac{1}{\cos^2 x} - \frac{1}{\sin^2 x} = \frac{\sin^2 x - \cos^2 x}{\cos^2 x \sin^2 x} \right)</math></p> <p><math>\text{RHS} = \frac{\sin^2 x}{\cos^2 x} - \frac{\cos^2 x}{\sin^2 x} = \frac{\sin^4 x - \cos^4 x}{\cos^2 x \sin^2 x} = \frac{(\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x)}{\cos^2 x \sin^2 x}</math>  <math>= \frac{\sin^2 x - \cos^2 x}{\cos^2 x \sin^2 x}</math>  <math>= \text{LHS} *</math> or equivalent</p>	<p>M1 A1                      A1 (3)                      M1                      A1                      A1 (3)</p>